

but the unsatisfactory nature of this mode of treatment is shown by the fact that in all modern books on trigonometry an angle is represented as generated by motion round an axis in a definite direction.

There are thus three geometrical quantities having direction, and the more than magical power of the method of Quaternions resides in the spell by which these three orders of quantities are brought under the sway of the same system of operators.

The secret of this spell is twofold, and is symbolised by the vine-tendril and the mason's rule and square. The tendril of the vine teaches us the relation which must be maintained between the positive direction of translation along a line and the positive direction of rotation about that line. When we have not a vine-tendril to guide us, a corkscrew will do as well, or we may use a hop-tendril, provided we look at it not directly, but by reflexion in a mirror.

The mason's rule teaches us that the symbol, as written on paper, is not a real line, but a mere injunction, commanding us to measure out in a certain direction a vector of a length so many times that of the rule. Without the rule the symbol would have no definite meaning. Thus the rule is the unit of the Quaternion system, while the square reminds us that the right angle is the unit versor.

The doctrine of the unit is a necessary part of every exact science, but in Quaternions the application of the same operators to versors, vectors, and areas is utterly unintelligible without a clear understanding of the function of the unit in the science of measurement.

Whether, however, it is better to insinuate the true doctrine into the mind of the student by a graduated series of exercises, or to inculcate it upon him at once by dogmatic statements, is a question which can only be determined by the experience of a new generation, who shall have been born with the extraspatial unit ever present to their consciousness, and whose thoughts, guided by the vine-tendril along the Quaternion path, shall turn always to the right hand, and never to the left.

Prof. Kelland tells us in the preface to the work to which we have alluded that, whereas Sir W. R. Hamilton and Prof. Tait have written treatises on Quaternions for mathematicians, the time has come when it behoves some one to write for those who desire to become mathematicians. Whatever, therefore, advanced mathematicians may think of this book, they ought to reserve their judgment as to its difficulty till they have ascertained how it is assimilated by those for whom it is written—those in whom the desire to become mathematicians has not yet become alloyed with the consciousness that they are mathematicians. For while Prof. Kelland—as he has elsewhere told us—finds but little difficulty in teaching the elements of the doctrine of Vectors to his junior classes, Hamilton himself, the great master of the spell, when addressing mathematicians of established reputation, found, for his Quaternions, but few to praise and fewer still to love.

Prof. Kelland, by the clearness and orderliness of his statements, and by boldly getting rid of everything which is unnecessarily abstruse, has done more than any other man towards rendering the subject easy to the student, and reconciling even the case-hardened mathematician to

the new method, as applied to geometrical questions of old-established truth.

The other aspect of Quaternions, as a method which every mathematician *must* learn in order to deal with the questions which the progress of physics brings every day into greater prominence, is hinted at by Prof. Tait in the last chapter of the book. He there introduces us to the linear and vector function⁷ of the first degree under its kinematical aspect of a homogeneous strain. The importance of functions of this kind may be gathered from the fact that a knowledge of their properties supplies the key to the theory of the stresses as well as the strains in solid bodies, and to that of the conduction of heat and electricity in bodies whose properties are different in different directions, to the phenomena exhibited by crystals in the magnetic field, to the thermo-electric properties of crystals, and to other sets of natural phenomena, one or more of which the scientific progress of every year brings before us.

But as we believe that Prof. Tait is about to bring out a new edition of his treatise on Quaternions, in which this higher aspect of the subject will be brought more prominently forward, we reserve our remarks on Quaternions as an instrument of physical research till we have the subject presented to us by Prof. Tait in a form which adequately represents its latest developments.

MARKHAM'S "UNKNOWN REGION"

The Threshold of the Unknown Region. By Clements R. Markham, C.B., F.R.S., Secretary of the Royal Geographical Society, formerly of H.M. Arctic ship *Assistance*. (London: Sampson Low and Co., 1873).

HE must be a sorry story-teller who manages to make a traveller's tale uninteresting, especially if the traveller be a voyager, and still more if his voyages have led him into unknown regions. Of all forms of narrative we think it will be generally acknowledged that narratives of discovery are by far the most popular, as is testified by the abundance of this kind of literature, historical and fictitious, provided for the delectation of the young. No doubt this may be largely accounted for by the fact that a discoverer of new lands is continually unveiling the unknown to those who listen to his tale, thereby appealing to one of the strongest and most fruitful characteristics of the human mind, that of curiosity. Every step taken by a discoverer, every knot sailed by his "good ship," we know will lead him among fresh wonders. Once upon a time the Unknown Region—that is, the region unknown to those peoples who have had a thirst for knowledge to any fruitful extent—was in sooth wide enough, when first our Aryan forefathers left their eastern home, and had "all the world before them where to choose." Even four centuries ago the greater part of the earth waited the coming of the European descendants of those primitive discoverers who first turned their faces eagerly and inquisitively to the unknown west. But ever since then the boundary of the Unknown Region has been gradually pushed farther and farther back, until now there remains comparatively little to be found out in order to enable geographers to complete the configuration of the lands of the globe. The extent of our dwelling-place is now pretty well known, though there is yet abundance of

work for many generations of explorers ere the contents of land and water be anything like fully disclosed.

Of all narratives of discovery, those relating to Arctic regions bear, in our estimation, the palm for intensity of interest, and we are sure there are many who think along with us in this matter. It would be difficult to say briefly why this is so. It may be mainly that there the mystery of the unknown, so far as relates to the surface of our globe, is concentrated. No doubt, also, there is a weird fascination around those eerie, rugged, ice-bound regions of the far north, which have been the scene of a greater number of deeds of heroic daring for noble and disinterested purposes, than any other region of the globe of equal extent. There is also a general though perhaps vague, yet we believe, well-founded belief, that within these regions lie solutions to many of the yet mysterious problems of science; that if once all the phenomena that lie within the yet unlifted veil were exposed and understood, they would afford us the means of tracing with something like certainty the history of our earth through many geological ages. In more senses than one, we are there on the threshold of the unknown.

There is somehow not the same attractiveness about Antarctic exploration, though, as Dr. Neumayer has well shown, it is certainly calculated to yield valuable results to Science, and indeed has already done so. This may partly arise from the scarcity in these regions of land and of life of all kinds, which are abundant enough in certain regions of the known north. Indeed, the tract around which the interest of Arctic discovery is concentrated may be regarded as but a continuation of the great American Continent.

We are sure that all who read this immensely interesting volume of Mr. Markham's will agree with what we have said. No more attractive subject for a work exists than the history of Arctic discovery; no man knows this subject better than Mr. Markham; and few could have written a volume on the subject more full of interest and of valuable information clearly arranged than the one before us. The object of the volume, Mr. Markham tells us, "is to give the public a correct knowledge of the whole line of frontier separating the known from the unknown region round the North Pole, to recall the stories of early voyagers, to narrate the recent efforts of gallant adventurers of various nationalities to cross the threshold, to set forth the arguments in favour of a renewal of Arctic exploration by England, and to enumerate, in detail, the valuable and important results to be derived from North Polar discovery." Mr. Markham's main design is evidently to show that the only certain gateway to the Pole is by the Smith Sound route, and this design he accomplishes in a way that cannot fail to convince any unprejudiced reader, by going over the whole story of Arctic discovery from the time that that hardy Norseman Lief, the son of Eric the Red, in 1001, made his abortive discovery of North America, down to the present year, when the world was astounded by the news of the discoveries and adventures of the ill-equipped but remarkably successful *Polaris* expedition. One's blood is once more stirred by the story of these fearless early English and Dutch adventurers, Burrough and Pett and Jackman and Barentz and Hudson and others, who dared to face the dangers of Arctic navigation in mere "cock-

boats" of 20 and 40 and 80 and 100 tons. The story of Barentz and his companions especially is told with considerable fulness, and it is with a very strange kind of feeling that one reads of the discovery, in 1871, of the very hut in which these stout-hearted Dutchmen passed the winter of 1596-7, and goes over the long catalogue of "Barentz relics" found therein.

Mr. Markham recounts the principal attempts that have been made to pierce through the formidable barrier of ice that guards the North Pole. "There are three approaches by sea to this land-girt end of the earth: through the wide ocean between Norway and Greenland, through Davis' Strait, and through Behring's Strait—one wide portal and two narrow gates." At present no one seems to think of attempting the last-mentioned route, advocates of Arctic exploration being divided between the Spitzbergen route, as the wide sea between Greenland and Novaya Zemlya is called, and the Smith Sound route, the route through the winding passages that lie between Greenland and the American continent. Mr. Markham, in considerable detail, recounts the various expeditions which, from the days of Barentz down to our own time, have charged the barrier that hems the Pole between the east coast of Greenland and Novaya Zemlya. He states with perfect fairness and with all necessary fulness the progress made by each expedition, and the invariable result, so far as the attempt to approach the Pole is concerned, has been failure. The highest latitude attained by this route was that reached by the well-equipped sledge expedition of Parry in 1827, 82° 45' N.; but the difficulties which the expedition had to encounter were so stupendous as, when combined with what is known of the conditions which influence the movements of the pack in this direction, to utterly forbid any hope of attaining the desired goal by the Spitzbergen route. The inevitable conclusion to be derived from the many fruitless attempts which have hitherto been made by this route is, "that by the Spitzbergen route, in a bad season, nothing whatever can be done; and in a favourable season a steamer may possibly press one or two, or even more degrees farther north than has hitherto been reached, and obtain some valuable deep-sea soundings and temperatures, but no other scientific results in the absence of land. The Spitzbergen route cannot be recommended, because there is no sure prospect of exploring an extensive unknown area, and because no valuable results in geology, botany, ethnology, or geodesy could be obtained under any circumstances." On this point Arctic authorities are all but unanimous, as they are also on the point that by the Smith Sound route a well-equipped Government expedition, if sent out next spring, would be almost certain to return within three years with the mystery of the "Polynia" cleared up, and with results in nearly all departments of Science not only invaluable from a purely scientific point of view, but of the highest practical importance. The very last attempt that has been made by the Smith Sound route seems to us to prove triumphantly that it would at present be folly to attempt to reach the pole by any other route, and that if the meagrely equipped and badly disciplined *Polaris* expedition accomplished so much in a very few days, an expedition such as Government will, we hope, feel bound to send out, will be sure to accomplish the remaining 400 or 500 miles

that lie between Hall's farthest north point and the Pole.

True, there are a few unhealthy croakers, as there always have been, and will be, we fear, for many generations to come, who ask What is the good of incurring so much danger and expense, for the mere gratification of curiosity, or, at best, to satisfy the wishes of a few men of science? But we feel confident that the great body of the English people will ask no such questions, but would hail with enthusiasm the decision of the Government to crown the glory which England has hitherto gained in Arctic exploration by sending out one more expedition whose task it would be to return with the long-sought-for secret in its keeping. It is beginning to dawn upon the ordinary English mind that, after all, the apparently unpractical researches of scientific men are frequently pregnant with results of the most important practical bearing on the welfare of the country and the race.

As for the element of danger, Mr. Markham convincingly shows by unimpeachable statistics, that the loss of men by the Smith Sound route, from causes connected with the climate and the peculiarities of the service, is almost incredibly small. One of the most distinguished medical officers who has served in the Arctic regions declares, that "of all seas visited by men-of-war the Arctic have proved the most healthy. . . . The risk by climate and disease which is run in a voyage to the Arctic seas—such as a Royal Expedition necessitates—is not greater than that which a ship like the *Challenger* will incur in her voyage of discovery." The dangers, or rather difficulties, which have to be faced are only such as brave men are eager to confront, and the service is one which our naval officers and men glory in in time of peace, and is certainly an infinitely better use to put them to than to keep them idling at home or on foreign stations. As to the question of expense, the article in a recent number will show that the less said by Government on this score the better.

All these and many other points in connection with Arctic exploration will be found fully and clearly discussed in Mr. Markham's volume, in which the invaluable results, scientific and practical, in nearly all departments of Science to be obtained from a Government Expedition are set forth with great fulness, clearness, and force. The volume concludes with an account of the interview that took place last year between the Arctic deputation and Mr. Lowe, the result of which was such as to give good grounds for expecting that this year Government will feel bound to organise an adequate expedition to leave our shores next spring to find its way to the Pole by the Smith Sound route.

The numerous maps by which the volume is illustrated are beautifully drawn, and are of the greatest assistance in enabling the reader to understand the interesting story of Arctic discovery so well told by Mr. Markham. As a mere story the work is a masterly one; and if anyone wants to know within short space what has already been done in the discovery of the Arctic regions, what still remains to be done, and what results are to be expected from further exploration, he could not do better than read Mr. Markham's "Threshold of the Unknown Region."

OUR BOOK SHELF

Annual Record of Science and Industry for 1872; edited by Spencer F. Baird, with the assistance of eminent men of Science. (New York: Harper and Brothers, 1873.)

THE praise which we were able to bestow on the first of Prof. Baird's Annual Records, that for 1871, can be fully repeated with regard to its successor. The only method of "reviewing" a work of this kind, is to refer in general terms to its scope, and to the degree to which the compiler appears to have fulfilled the promises of his programme. On these points we can speak in the most favourable terms. As far as a cursory glance through the pages of the volume enables us to speak, we believe that purchasers of the book will find it a most useful addition to their library shelves. The paragraphs refer to the most noteworthy additions to scientific knowledge or observation made during the year, and have been compiled with commendable terseness and perspicuity from a large range of English, American, and Continental sources. A carefully *raisonné* table of contents, and an alphabetical index, will enable the student to turn without difficulty to any desired subject. Although absolute freedom from errors, typical and otherwise, can hardly be expected in a work with so large a scope, the American "Record" contrasts most favourably in this respect with some similar volumes published in this country. We do not know where to find a more complete record of the science of the year; and we shall hope to see a long series of these useful volumes.

The Borderland of Science. By Richard A. Proctor. (London: Smith, Elder, and Co. 1873.)

THESE Essays are reprinted from the *Cornhill Magazine*. The titles are as follows:—"The Herschels and the Star-Depths;" "A Voyage to the Sun;" "A Voyage to the Ringed Planet;" "A Giant Planet;" "Life in Mars;" "A Whewellite Essay on the Planet Mars;" "Meteors—Seed-bearing, and otherwise;" "A Recent Star-shower, and Star-showers generally;" "News from the Moon;" "Earthquakes;" "The Antarctic Regions:" "A Few Words about Coal;" "Notes on Flying and Flying-Machines;" "Gambling Superstitions;" "Coincidences and Superstitions;" "Notes on Ghosts and Goblins."

Sommario delle Lezioni di Fisica, date dal Professore Enrico Dal Pozzo di Mombello, nella Libera Università di Perugia. (Foligno: Pietro Spariglia, 1873.)

GANOT'S Treatise on Physics has been translated into Italian and is no doubt largely used in the country; also in 1870 Prof. Cantoni, of Milan, published a course of Physical Lectures. The work before us by Prof. Dal Pozzo is to some extent based upon that of Cantoni; it is a summary of two courses of lectures delivered in the free University of Perugia. The University (founded in 1307) is one of the oldest in Europe, and possesses a good library, botanical gardens, and mineralogical collections. We cannot at this moment call to mind any scientific associations connected with the place, as with Pisa, Bologna, and Pavia. The town itself has been mentioned any time for two and twenty centuries, and it is a noted school of music.

We can scarcely judge of the science of Perugia from the work before us. The students must be very clear-headed men if they can follow Prof. Pozzo's arrangement. It is certainly most novel. It may have its advantages. He begins cleverly enough with an account of the "Energy of the Universe," embracing some general properties of bodies, actual and potential energy, conservation and dissipation of energy. The author uses the terms *forza attiva* and *forza di posizione*, in place of our more usual terms. We are glad to find him acquainted with the works of Thomson, Balfour Stewart